UNIVERSITY OF SASKATCHEWAN
College of Engineering

CE 418.3 Reinforced Concrete I
FINAL EXAM

Time: 3 Hours | December 10, 1997

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Note: Students are permitted to use the CPCA Concrete Handbook, including CSA Standard A23.3-94.

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Marks

1. A symmetrically placed 500 mm x 500 mm column can be considered to be pinned at the interface with a 4 m x 4 m x 850 mm deep footing. The column transfers specified loads of $P_D = 2000$ kN and $P_L = 1500$ kN to the footing. All concrete has $f'_c = 25$ MPa (normal density with a unit weight of 2400 kg/m$^3$) and all reinforcing steel is grade 400.

10 a) Assuming the soil bearing capacity is adequate, check two way (punching) shear. Do *not* change the footing depth if the punching shear is inadequate, and do *not* check one way (beam) shear.

5 b) Design the dowels required at the column/footing interface; however, do not consider anchorage (development or splicing) requirements for the dowels.

10 c) Using the CPCA Handbook design aids for moment, determine the flexural reinforcing steel required in the footing. Do not consider anchorage requirements for this steel.

5 d) If the overburden is 1 meter deep (density = 1600 kg/m$^3$) over the 850 mm footing, determine the allowable bearing capacity that is needed in the soil below the footing.

2. A rectangular reinforced concrete beam has b x d = 300 mm x 485 mm, and the material properties are $f_y = 400$ MPa and $f'_c = 30$ MPa (light weight concrete).

12 a) Calculate the spacing of 1/4 inch (6.35 mm) diameter plain stirrups required at a point where the factored shear load $V_f = 120$ kN.

3 b) Determine the maximum factored shear resistance for the beam in Part (a) in a region where stirrups are not provided.

3. An equilateral triangular column cross section (Fig. 1) has $f'_c = 30$ MPa (normal density concrete and is reinforced with 3 - No. 25 bars ($f_y = 300$ MPa). Small diameter confinement bars (not shown in the figure) and ties provide the necessary confinement to the concrete to satisfy A23.3-94 requirements for columns; however, these confinement bars are *not* to be considered in any calculations. Any bending is to be considered about the y-y axis only.

10 a) Find the location of the plastic centroid (P.C.), as measured by the distance $x_o$ from the center of the No. 25 bars. For $f'_c = 30$ MPa, $\alpha_1 = 0.805$ and $\beta_1 = 0.895$.

18 b) Determine the balanced condition factored axial load and moment, and the eccentricity distance ($e_b$) as measured from the P.C., that the moment corresponds to.

7 c) Determine the factored axial load and moment which results in no strain in the 3 - No. 25 bars.

20 d) If a factored compressive load of 1200 kN is applied to the column to the right of the plastic centroid location found in Part (a), calculate the neutral axis distance "c" that would be needed ~~for finding the accompanying factored~~ moment that is permitted with the compressive load. However, in the interests of time, do not calculate this moment.

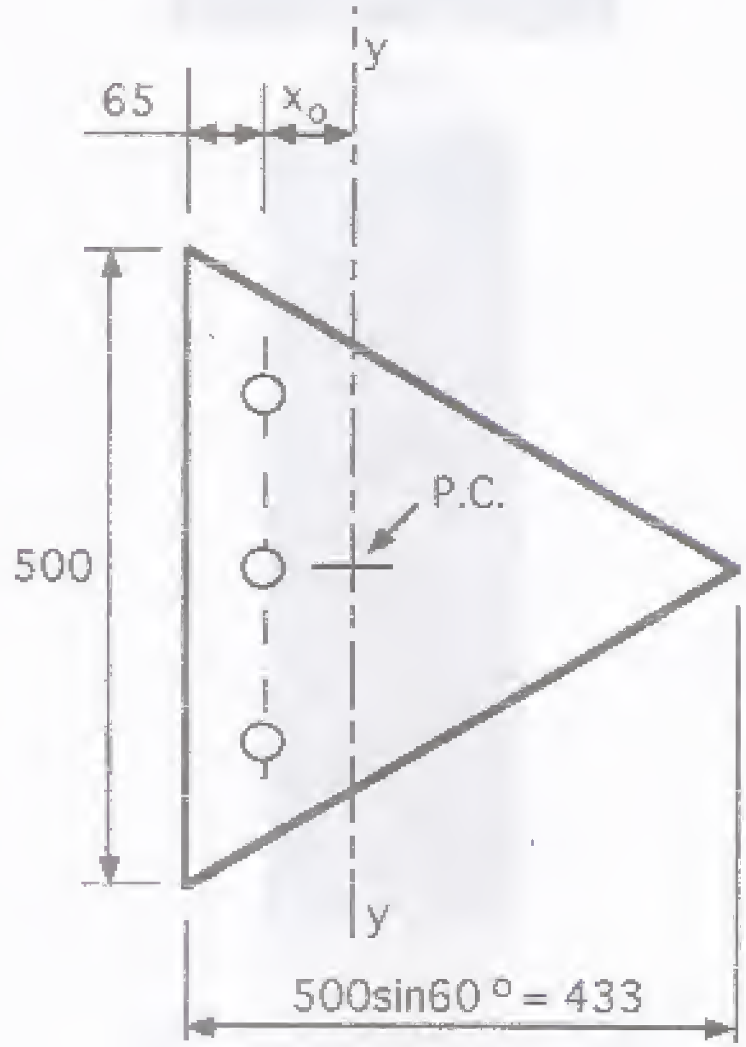


Fig. 1

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3 b) Determine the maximum factored shear resistance for the beam in Part (a) in a region where stirrups are not provided.

(a) $V_c$ (WITH MIN STIRRUPS) $= 0.2\lambda\phi_c\sqrt{f'_c}\,b_w d = 0.2\times0.75\times0.6\sqrt{30}\times300\times485 = \underline{71.72\times10^3 N}$ (Cl. 11.3.5.1) } 2%

Cl. 8.6.5 (low density conc) → 0.85 (for semi-low density)?

$V_s = V_f - V_c = 120 - 71.72 = \underline{48.28\,kN}$ } 1%

STRENGTH (Cl. 11.3.7) $s = \dfrac{\phi_s A_v f_y d}{V_s} = \dfrac{0.85\times2\overbrace{\left(\pi\times\frac{6.35^2}{4}\right)}^{31.67}\times400\times485}{48.28\times10^3} = \underline{216.4\,mm}$ (2 legs/stirrup) } 3%

MAX PERMITTED SPCG (Cl. 11.2.11) $V_f = \underline{120\times10^3 N} < 0.1\lambda\phi_c f'_c b_w d = 0.1\times0.75\times0.6\times30\times300\times485 = \underline{196.4\times10^3 N}$

$\therefore S_{MAX} = 0.7d \not> 600\,mm = 0.7\times485 = \underline{339.5\,mm} > \underline{216.4\,mm}$ – O.K. } 2%

CHECK MAX $V_f$ FOR SIMPLE APPROACH (Cl. 11.3.4) $V_f \le V_c + 0.8\lambda\phi_c\sqrt{f'_c}\,b_w d = 71.72\times10^3 N + 4(71.72\times10^3 N)$

$= 5(71.72\times10^3 N) = \underline{358.6\times10^3 N}$ } 2%

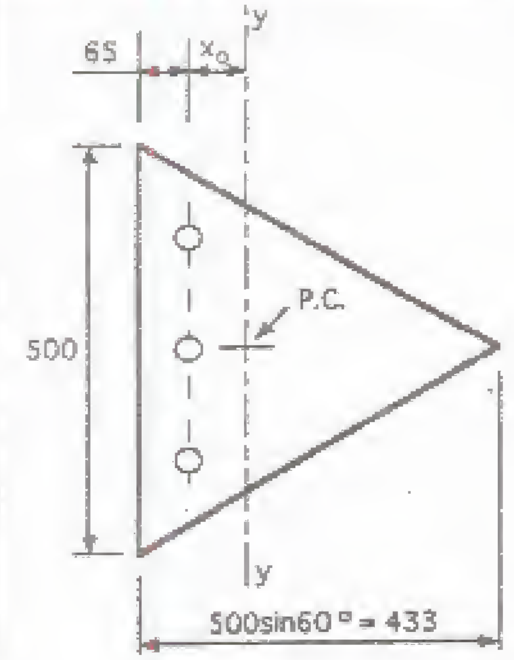
$\underline{120\,kN \le 358.6\,kN \rightarrow O.K.}$

CHECK $A_{v\,MIN}$ (Cl. 11.2.8.4) $A_v \ge 0.06\sqrt{f'_c}\,\dfrac{b_w S}{f_y}$ ; $2(31.67) \ge \dfrac{0.06\sqrt{30}\times300\times216.4}{400}$ } 2%

$\underline{63.34\,mm^2} > \underline{53.34\,mm^2} \rightarrow$ O.K.

(b) $V_f = V_c + \cancel{V_r}^{0} = V_c = \dfrac{260}{1000+d}\lambda\phi_c\sqrt{f'_c}\,b_w d \not< 0.10(\quad)$ Cl. 11.3.5.2 } 3%

$= \underbrace{\dfrac{260}{1000+485}}_{0.1751 > 0.10 \leftarrow \text{CONTROLS}}\times0.75\times0.6\sqrt{30}\times300\times485 = \underline{62.8\times10^3 N}$




3. An equilateral triangular column cross section (Fig. 1) has $f'_c$ = 30 MPa (normal density concrete and is reinforced with 3 - No. 25 bars ($f_y$ = 300 MPa). Small diameter confinemer bars (not shown in the figure) and ties provide the necessary confinement to the concrete to satisfy A23.3-94 requirements for columns; however, these confinement bars are *not* to be considered in any calculations. Any bending is to be considered about the y-y axis only.

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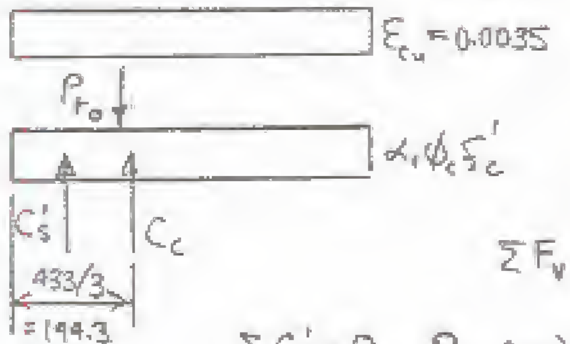



(a) $C_c = \alpha_1 \phi_c f'_c \left(\frac{1}{2} \times 500 \times 433\right)$

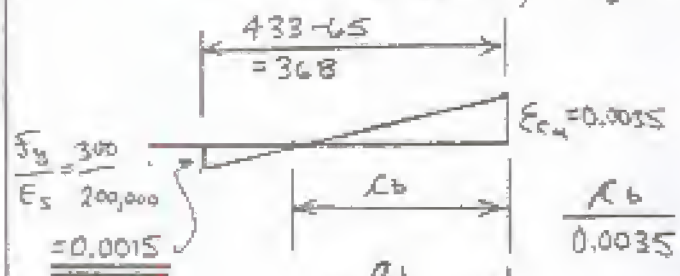
$= 0.805 \times 0.60 \times 30 (108250) = \underline{1568.5 \times 10^3 N}$

$C'_s = A'_s(\phi_s f_y - \alpha_1 \phi_c f'_c)$

$= 3 \times 500 \overbrace{(0.85 \times 300 - 0.805 \times 0.60 \times 30)}^{240.51} = \underline{360.8 \times 10^3 N}$

$\Sigma F_y = 0;\ P_{ro} = 1568.5 + 360.8 = \underline{\underline{1929.3\ kN}}$

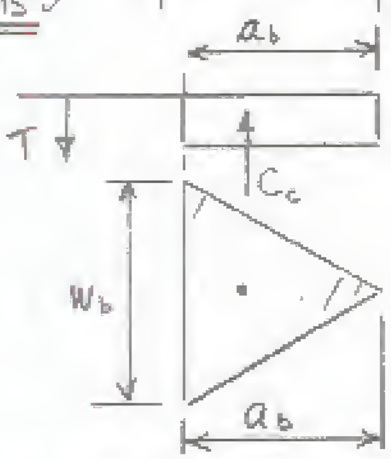
$\Sigma C'_s = 0;\ P_{ro}(x_o) - C_c \underbrace{\left(\frac{433}{3} - 65\right)}_{79.33} = 0;\ x_o = \frac{1568.5}{1929.3}(79.33) = \underline{\underline{64.50\ mm}}$




(b) $\underline{P_b, M_b \ \& \ e_b}$

$\frac{c_b}{0.0035} = \frac{433-65}{0.0035+0.0015}$ ; $c_b = \frac{0.0035}{0.0050}(368) = \underline{\underline{257.6\ mm}}$

$a_b = \beta_1 c_b = 0.895 \times 257.6 = \underline{\underline{230.55\ mm}}$




$C_c = \alpha_1 \phi_c f'_c \left(\frac{1}{2} W_b a_b\right)$ ; $W_b = \frac{500}{433} \times a_b = \frac{500}{433} \times 230.55 = \underline{\underline{266.2\ mm}}$

$C_c = 0.805 \times 0.60 \times 30 \left(\frac{1}{2} \times 266.3 \times 230.55\right) = \underline{444.7 \times 10^3 N}$

$T = 1500 \times 0.85 \times 300 = \underline{382.5 \times 10^3 N}$ $\therefore P_{rb} = C_c - T$

$= 444.7 - 382.5 = \underline{\underline{62.2\ kN}}$

$\Sigma M_T = 0$

$C_c(433 - 65 - \frac{2}{3}\overset{230.55}{a_b}) - P_{rb}(e_b + x_o) = 0$

$C_c(214.3) - \underbrace{P_{rb} e_b}_{M_{rb}} - P_{rb} x_o = 0$ $\therefore M_{rb} = 444.7 \times 10^3 (214.3) - 62.2 \times 10^3 (64.5)$

$= 95.3 \times 10^6 - 4.01 \times 10^6 = \underline{\underline{91.3 \times 10^6\ N \cdot mm}}$

$e_b = \frac{M_{rb}}{P_{rb}} = \frac{91.3 \times 10^6}{62.2 \times 10^3} = \underline{1468\ mm}$

(OR) $\Sigma M_{x_o\,(P.C.)} = 0$

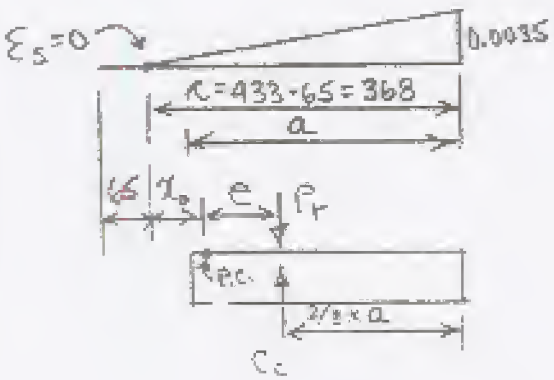
$T(x_o) + C_c(368 - x_o - \frac{2}{3}a_b) - \overset{P_{rb} e_b}{M_b} = 0$

$\therefore M_{rb} = 382.5 \times 10^3 (64.5) + 444.7 \times 10^3 \underbrace{(368 - 64.5 - \frac{2}{3} \times 230.55)}_{149.8}$

$M_{rb} = 24.67 \times 10^6 + 66.61 \times 10^6 = \underline{\underline{91.3 \times 10^6\ N \cdot mm}}$ $\therefore e_b = \frac{91.3 \times 10^6}{62.2 \times 10^3} = \underline{\underline{1468\ mm}}$

S0 a = 2

(c)




$$a = \beta_1 c = 0.895 \times 368 = \underline{329.4\ mm}\ ; \quad C_c = 0.805 \times 0.60 \times 30 \underbrace{(a \times W \times \tfrac{1}{2})}_{= 1.155a}$$

$$\underline{\underline{C_c = 907.5 \times 10^3 N}} \equiv P_r$$

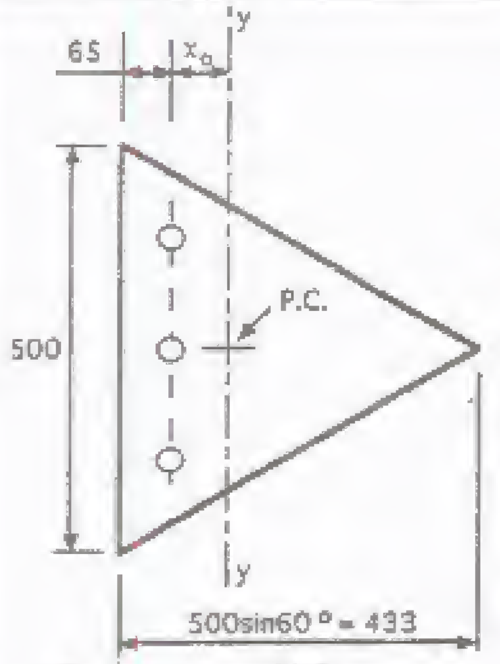
$$M_r = P_r e = C_c (368 - \tfrac{2}{3} a - x_o) = 907.5 \times 10^3 (\underbrace{368 - \tfrac{2}{3} \times 329.4 - 64.5}_{83.9})$$

$$= 907.5 \times 10^3 (83.9) = \underline{\underline{76.1 \times 10^6 N \cdot mm}}$$

PROBLEMS

CE418.3 R.C.I CLASS

NAME T.R. DATE DEC 10, 1997

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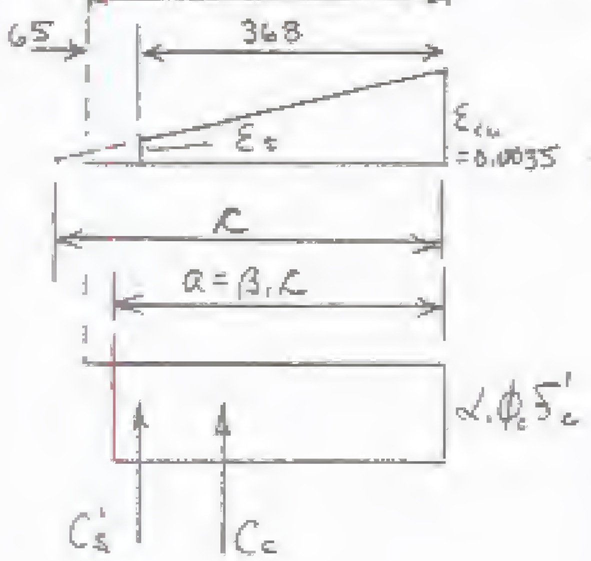
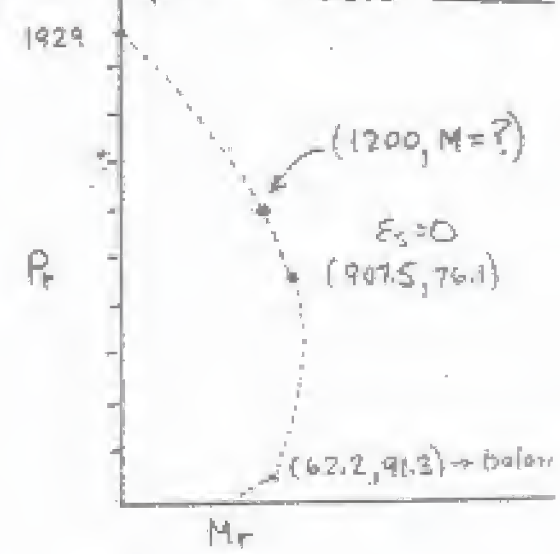
(d) For $P_r = 1200\,kN > P_{r(\varepsilon_s=0)} = 907.5\,kN$
$< P_{r_o} = 1929\,kN$ } ∴ Steel will be in compression.

- Steel probably won't yield → assume & verify
- also, assume "a" < 433 mm $\left(c < 433/\beta_1 = 433/0.895 = \underline{483.8\,m}\right.$
> 368 mm




STRAINS

$$\frac{\varepsilon_s}{0.0035} = \frac{c-368}{c}$$

$$f_s = E_s\varepsilon_s = 200{,}000 \times 0.0035\left(\frac{c-368}{c}\right)$$
$$= 700\left(\frac{c-368}{c}\right)$$

EQUILIB: $\underline{\Sigma F_y = 0}$ ; $C_c + C'_s - 1200\times10^3\,N = 0$

$$\alpha_1\phi_c f'_c\left(a\times\frac{500}{433}\times a\times\frac{1}{2}\right) + A_s\left(\phi_s f_s - \alpha_1\phi_c f'_c\right) - 1200\times10^3 = 0$$

$$0.805\times0.6\times30\left(0.895c\times\frac{500}{433}\times0.895c\times\tfrac{1}{2}\right) + 1500\left[\overbrace{0.85\times700}^{595}\left(\frac{c-368}{c}\right) - \overbrace{0.805\times0.60\times30}^{14.49}\right] - 1200\times10^3 = 0$$

$$6.701c^2 + 1500\left[580.51 - \frac{218.96\times10^3}{c}\right] - 1200\times10^3 = 0$$

$$6.701c^3 + 870.8\times10^3c - 328.44\times10^6 - 1200\times10^3c = 0$$

$$6.701c^3 - 329.2\times10^3c - 328.44\times10^6 = 0$$

$$\underline{c^3 - 49.13\times10^3c - 49.01\times10^6 = 0}$$

Try

$c = 433$ $= +10.9\times10^6$

400 $= -4.66\times10^6$

410 $= -0.23\times10^6$

... = [illegible]

411

410.5

$= + 0.2270$

$= -0.0046 \times 10^6 \rightarrow f_s = 700 \left(\frac{410.5 - 368}{410.5}\right) = 72.5\ MPa < f_y = 300\ M$

O.K.

$a = 0.895 \times 410.5 = \underline{367.4\ mm} \approx 368\ mm$

{ COMPENSATION FOR HOLES O.K.

OR → COULD HAVE USED $\phi_c\, \varepsilon_c\, E_c$ }

$\varepsilon_s = 0.0035 \left(\frac{c - 368}{c}\right)$